Answer all questions

No	Sub Sec	Description	Marks
1.	(a)	Without constructing truth table, find the principal conjunctive normal form of $(p \rightarrow (q \wedge r)) \wedge (\sim p \rightarrow (\sim q \wedge \sim r))$.	(
	(b)	Derive $a \rightarrow (b \rightarrow d)$ using the Rule of Conditional Proof (CP-rule) from the premises $a \rightarrow (\neg b \vee c)$ and $b \rightarrow (c \rightarrow d)$	(
2.	(a)	Show that the hypotheses "If you send me an e-mail message, then I will finish writing the program", "If you do not send me an e-mail message, then I will go to sleep early", and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."	(
	(b)	Determine whether these system specifications are consistent: "The diagnostic message is stored in the buffer or it is retransmitted", "The diagnostic message is not stored in the buffer", "If the diagnostic message is stored in the buffer, then it is retransmitted".	(
3.	(a)	Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book"	(6
	(b)	Show that q can be derived from the premises using indirect method $p \rightarrow q, r \rightarrow q, s \rightarrow (p \vee r)$.	(4)
4.	(a)	If α and β are elements of the symmetric group S_4, given by $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$. Find $\alpha\beta$, $\beta\alpha$ and α^{-1}.	(3)
	(b)	Obtain all the distinct left cosets of $\{[0], [3]\}$ in the group $(Z_6, +_6)$.	(2)

Aug 2024

CIHTCIHTCI

	(c)	If $S = N \times N$, the set of ordered pairs of positive integers with operation $*$ defined by $(a, b) * (c, d) = (ad + bc,\ bd)$. Show that $(S, *)$ is a semigroup. And, if $f: (S, *) \to (Q, +)$ is a function defined by $f(a, b) = \frac{a}{b}$, show that f is homomorphism.	(5)
5	(a)	Find the code words generated by the encoding function $e: B^2 \to B^5$ with respect to the parity check matrix $H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.	(6)
	(b)	Prove that the set $G = \{1, 2, 3, 4\}$ is abelian group with respect to ordinary multiplication modulo 5.	(4)

********** All the best **************

VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

Reg. Number: 22BCE1344

Continuous Assessment Test (CAT) – I August 2024

Programme	:	B.Tech.	Semester	:	FALL 2024-2025
Course Code & Course Title	:	BMAT205L Discrete Mathematics and Graph Theory	Slot	:	C2+TC2+TCC2
Faculty	:	Prof. Aarthy B Dr. Amit Kumar Rahul Prof. Anitha G Dr. Ankit Kumar Dr. Padmaja N Dr. Poulomi De Dr. Surath Ghosh	Class Number	:	CH2024250102066 CH2024250102265 CH2024250102267 CH2024250102069 CH2024250102266 CH2024250102068 CH2024250102268
Duration	:	90 Minutes	Max. Mark	:	50

General Instructions:

- Write only your registration number on the question paper in the box provided and do not write other information.
- Use statistical tables supplied from the exam cell as necessary
- Use graph sheets supplied from the exam cell as necessary
- Only non-programmable calculator without storage is permitted

Answer all questions
(5×10=50)

Q. No	Sub Sec.	Description	Marks
1.	(a)	Without using truth table, find PDNF of $\neg(p \vee (\neg p \wedge \neg q \wedge r))$.	(5)
	(b)	Identify the bound variable, free variable and the scope of the following expression: $\forall x(P(x) \wedge Q(x)) \vee \forall y R(y)$. Also, write the converse, contrapositive and inverse of the following proposition symbolically and in words "If the weather is nice, then I'll wash the car".	(2+3)
2.	(a)	Prove that $\neg p \leftrightarrow q, q \rightarrow r, \neg r \Rightarrow p$ is valid.	(5)
	(b)	Show that the premises "An employee in my office has not completed his daily work" and "Everyone in my office completed his monthly files" imply the conclusion "Someone who completed his monthly files has not completed his daily work".	(5)
3.	(a)	Prove the following equivalences by proving the equivalences of the dual without using truth table: $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \equiv p \wedge q$	(5)

	(b)	Let $P(x)$ be the statement "x has visited universal studios" where the universe consists of the students at UCF. Express each of the following statements using quantifiers (i) Some students at UCF have not visited universal studios. (ii) Not all students at UCF have visited universal studios. (iii) No student at UCF has not visited universal studios.	(2+2+1)
4.	(a)	If $*$ is the binary operation on $S = Q \times Q$, the set of ordered pairs of rational numbers and given by $(a, b) * (c, d) = (ac, ad + b)$, i) Prove $(S, *)$ is a semi group. Is it commutative? ii) Find the identity element of S iii) Which elements, if any, have inverses, and what are they?	(6)
	(b)	Let $\mathbb{R} - \{0\}$ represents set of all nonzero real numbers and M denotes the set of all 2×2 invertible matrices over $\mathbb{R}$. Determine whether the following map is a homomorphism. If so, what is its kernel? Given the map $f: \mathbb{R} - \{0\} \rightarrow M$ defined by $f(a) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$.	(4)
5	(a)	Given the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ corresponding the encoding function $e: B^3 \longrightarrow B^6$ find the parity check matrix and use it to decode the following received words and hence find the original message. Are all the words decoded uniquely? (i) 111101 (ii) 100100 (iii) 111100 (iv) 010100	(5)
	(b)	In the group S_6, a permutation group over $\{1,2,3,4,5,6\}$ $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 5 & 3 & 6 \end{pmatrix}$, $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 6 & 1 & 2 & 4 \end{pmatrix}$ determine α^{-2} and $x \in S_6$ such that $\alpha x = \beta$.	(1+2)
	(c)	Provide a justification for why $U(8) = \{1,3,5,7\}$, under multiplication modulo 8, is not a cyclic group.	(2)
**********All the best *************			

23BAI1038

VIT

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

Continuous Assessment Test I - August 2024

Programme:	B.Tech.	Semester :	FALL 2024-25
Course :	Discrete Mathematics and Graph Theory	Code :	BMAT205L
		Slot :	D1+TD1+TDD1
Faculty :	Dr. Kalyan Manna, Dr. Nathiya, Dr. Jayagopal, Dr. Saurabh Chandra Maury, Dr. Padmaja, Dr. Dhivya P, Dr. Berin Greeni, Dr. Poulomi De, Dr. Ganaprasanna	Class ID :	CH2024250102070, 2071, 2072, 2074, 2075 2076,2079,2080, 2081
Time :	90 Minutes	Max.Marks :	50

Answer all the Questions ($5 \times 10 = 50$)

1. (a) Determine whether the following argument is valid or not: If the lecture proceeds, then either black board is used or the slides are shown or the tablet pc is used. If the black board is used, then students are not comfortable in reading the black board. If the slides are shown, then students are not comfortable with the speed. If the tablet pc is used, then it causes a lot of disturbances to the instructor. The lecture proceeds and the students are comfortable. Therefore, the instructor faces disturbances. [7]

   (b) Write the symbolic representation and the negation of "Even though it is not raining, it is still cool". [3]

2. (a) Determine whether the following is a tautology or not by using truth table: $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$. [4]

   (b) Find the PDNF and PCNF of $p \vee (\neg p \rightarrow (q \vee (\neg q \rightarrow r)))$. [6]

3. Construct the decoding table for the group codes generated by the generator matrix $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$. Use the decoding table to decode the following received words: 11100, 11000, 10100, 10111 and 00111. [10]

4. (a) Consider the direct product $\mathbb{R} \times \mathbb{R}$ of the additive group of real numbers and the function $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x, y) = 2x - y$. Show that f is a homomorphism of groups. Also, find its kernel. [4]

   (b) Check if the set $\mathbb{Z}$ of all integers form a group under the operation defined as $x * y = x - y$ for every $x, y \in \mathbb{Z}$. [4]

   (c) Let S_4 be the symmetric group on 4 elements. Find a subgroup of S_4 whose order is 2. [2]

5. (a) Use indirect method to prove that the conclusion $\exists z Q(z)$ follows from the premises $\forall x(P(x) \rightarrow Q(x))$ and $\exists y P(y)$. [6]

   (b) Write the symbolic representation and the negation of the following statement: "Every student solves at least one problem in the tutorial sheet". [4]

VIT
Vellore Institute of Technology
CHENNAI

Continuous Assessment Test I - August 2024

Programme		B.Tech	Semester	:	FALL 2024-25
Course	:	Discrete Mathematics and Graph Theory	Code	:	BMAT205L
			Slot	:	D2+TD2+TDD2
Faculty	:	Dr. Kalyan Manna, Dr. Nathiya, Dr. Vidhya, Prof. Sumathi, Dr. Om Namha Shivay Dr. Pavithra, Prof. Sakthi Devi, Dr. Ashish Kumar Nandhi, Dr. Dhivya P, Dr. Berin Greeni, Dr. Radha, Dr. Sandip Saha.	Class ID	:	CH2024250102089, 2090, 2093, 2095, 2096 2098,2101,2102, 2191, 2195, 2197
Time	:	90 Minutes	Max.Marks :		50

Answer all the Questions ($5 \times 10 = 50$)

1. (a) Determine whether the following argument is valid or not: The meeting can take place if all members are informed in advance and there is quorum (a minimum number of members are present). There is a quorum if at least 20 members are present. Members would have been informed in advance if there was no strike. Therefore, if the meeting was cancelled, then either there were fewer than 20 members present or there was a strike [6]

   (b) Let p and q be two propositions stating "Land slide happened" and "Disaster team rescued people" respectively. Write the statements corresponding to the following propositions: [4]

   i) $\neg p \wedge q$

   ii) $p \leftrightarrow q$

   iii) $\neg q \rightarrow \neg p$

   iv) $p \vee \neg q$

2. (a) Write the inverse, converse and contrapositive statements of "He scored high unless he studied well." [4]

   (b) Find the PDNF and PCNF of $(\neg p \vee \neg q) \rightarrow (p \leftrightarrow \neg q)$. [6]

3. (a) Let $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ be the generator matrix. Find all code words. Also find the corresponding parity check matrix to decode the following received words: 101011, 110010, 101110, 100111, 001110 and 110101. [8]

   (b) Is it possible to find a subgroup of order 3 for the group $(Z_{13}, +_{13})$? If so, write the subgroup otherwise give reasons. [2]

4. (a) Show that $(Z_2 \times Z_2, +_2)$ is an abelian group with respect to addition module 2. [5]

(b) Consider the direct product $\mathbb{R} \times \mathbb{R}$ of the additive group of real numbers. Check whether the subset $H = \{(x, y) \in \mathbb{R} \times \mathbb{R}, \text{ such that } 2x + y = 0\}$ is a subgroup of $\mathbb{R} \times \mathbb{R}$ or not. [3]

(c) Check whether $T : (\mathbb{R}, +) \to (\mathbb{R}, +)$ defined by $T(x) = 2x^2 + 1$ is a group homomorphism or not. [2]

5. (a) Write symbolic representation and the negation of the following: [5]

   i) Every integer which is divisible by 10 is also divisible by 2.

   ii) Some integers which are divisible by 2 are also divisible by 5.

   (b) Prove $(\forall x)[p(x) \to q(x)], (\forall x)[r(x) \to \neg q(x)]$ implies $\forall x[r(x) \to \neg p(x)]$ [5]