10. Find the Fourier transform of $f(x) = \frac{e^{-ax}}{x}$ and use it to evaluate $\int_0^\infty \tan^{-1}\left(\frac{x}{a}\right) \sin(x)\, dx$.

[10] (CO3/K5)

11. If $u_n = \sin^2(3n+5) + (n-1)^3$, then find the Z-Transform of u_n

[10] (CO5/K5)

12. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = 1$ & $y_1 = 0$ by using Z-Transform.

[10] (CO5/K3)

BL-Bloom's Taxonomy Levels - (K1-Remembering,K2-Understanding,K3-Applying,K4-Analysing,K5-Evaluating,K6-Creating)

VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

CHENNAI

Final Assessment Test(FAT) - Apr/May 2025

Programme	B.Tech.	Semester	Winter Semester 2024-25
Course Code	BMAT102L	Faculty Name	Prof. Sudip Debnath
Course Title	Differential Equations and Transforms	Slot	C1+TC1+TCC1
		Class Nbr	CH2024250500798
Time	3 hours	Max. Marks	100

Reg. No

Instructions To Candidates

- Write only your registration number in the designated box on the question paper. Writing anything elsewhere on the question paper will be considered a violation.

Course Outcomes

CO1: Find solution for second and higher order differential equations, formation and solving partial differential equations.

CO2: Understand basic concepts of Laplace Transforms and solve problems with periodic functions, step functions, impulse functions and convolution.

CO3: Employ the tools of Fourier series and Fourier transforms.

CO4: Know the techniques of solving differential equations and partial differential equations.

CO5: Know the Z-transform and its application in population dynamics and digital signal processing.

Answer any 10 Questions (10 × 10 Marks)

01. Find the general solution of the differential equation $(D^2 - 2D + 1)y = 5e^{4x}$

[10] (CO1/K1)

02. a. Find the charge $q(t)$ on the capacitor in an LRC circuit when $L = 0.25$ henry(h), $R = 10$ Ohm (Ω), $C = 0.001$ farad(f)-, $E(t) = 0, q(0) = 10$ Coulombs(C), and $i(0) = 0$. (5 Marks)

b. Form the PDE by eliminating arbitrary function f from the relation $yz + zx + xy = f(z/(x+y))$. (5 Marks)

[10] (CO1/K3)

03. Find the general solution of the partial differential equation $z(z^2 + xy)(px - qy) = x^4$

[10] (CO1/K2)

04. a. Express $f(t) = \begin{cases} 0, & 0 < t < \pi/2 \\ sin(t), & \pi/2 < t \end{cases}$, interms of heaviside function and find the Laplace transform of $f(t)$. (5 Marks)

b. Obtain Fourier series of the function $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$. (5 Marks)

[10] (CO2/K1)

05. Find $L^{-1}\left(\frac{(s+1)e^{-\pi s}}{s^2+s+1}\right)$.

[10] (CO2/K2)

06. Solve the following initial value problem (IVP) using the Laplace transform: $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t;\ x(0) = 2\ and\ x'(0) = -1$

[10] (CO4/K3)

07. By using the Laplace transform, solve the partial differential equation $\frac{\partial u}{\partial x} + 6x\frac{\partial u}{\partial t} = 12x$, with $u(0,t) = 2$, for $t > 0$ and $u(x,0) = 2$, for $x > 0$.

[10] (CO4/K3)

08. Find the Fourier series of $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$. Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.$

[10] (CO3/K2)

09. Find the Fourier transform of the given Gaussian function $f(x) = 2e^{-3x^2}$.

[10] (CO3/K1)