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CHENNAI

23BEC1091

Final Assessment Test(FAT) - Apr/May 2025

Programme	B.Tech.	Semester	Winter Semester 2024-25
Course Code	BECE207L	Faculty Name	Prof. Thiripurasundari D
Course Title	Random Processes	Slot	D1+TD1
		Class Nbr	CH2024250501178
Time	3 hours	Max. Marks	100

Instructions To Candidates

- Write only your registration number in the designated box on the question paper. Writing anything elsewhere on the question paper will be considered a violation.

Course Outcomes

CO1: Compute the probability density functions for multiple random variables.
CO2: Perform transformation on multiple random variables and complex random variable
CO3: Interpret the random processes in terms of stationarity, statistical independence, autocorrelation.
CO4: Compute the power spectral density of the random signals
CO5: Interpret the effect of random signals on LTI systems output both in the time and frequency domain.
CO6: Design the Optimum linear systems for extracting signals in the presence of noise

Section - I

Answer all Questions (7 × 10 Marks)

01. Given the joint distribution function
$F_{X,Y}(x,y) = u(x)u(y)\left[1 - e^{-ax} - e^{-ay} + e^{-a(x+y)}\right]$
(i) Find the conditional density functions $f_X(x|Y=y)$ and $f_Y(y|X=x)$ **[8 marks]**
(ii) Are the random variables X and Y statistically independent? **[2 marks]**

[10] (CO1/K3)

02. Statistically independent, zero-mean random processes $\alpha(t)$ and $\beta(t)$ have autocorrelation function
$R_{\alpha\alpha}(\tau) = A_0 sin(2\pi\tau)$

$R_{\beta\beta}(\tau) = B_0 e^{-9|\tau/2|}$
respectively

(i) Find the auto correlation function of the sum $W_1(t) = \alpha(t) + \beta(t)$ **[3 marks]**
(ii) Find the auto correlation function of the difference $W_2(t) = \alpha(t) - \beta(t)$ **[3 marks]**
(iii) Find the cross correlation function of $W_1(t)$ and $W_2(t)$ **[4 marks]**

[10] (CO3/K3)

03. Let two random processes $X(t)$ and $Y(t)$ be defined by
$X(t) = A\cos(\omega_o t) + B\sin(\omega_o t)$
$Y(t) = B\cos(\omega_o t) - A\sin(\omega_o t)$
where A and B are random variables and ω_o is a constant.
(i) Find the cross correlation between $X(t)$ and $Y(t)$. **[6 marks]**
(ii) Is $X(t)$ and $Y(t)$ jointly WSS? If not, what is the condition to make it as WSS? **[4 marks]**

[10] (CO3/K4)

04. (a) A stationary random process $X(t)$ has the power spectral density
$S_{XX}(\omega) = \frac{24}{\omega^2+16}$
Find the mean square value of the process. **[5 marks]**
(b) Let $X(t)$ and $Y(t)$ be statistically independent random processes with zero mean and their autocorrelation functions

$R_{XX}(\tau) = \frac{1}{2\pi}$

$R_{YY}(\tau) = e^{-10|\tau|}$

A complex random process is given by $Z(t) = [X(t) + jY(t)]e^{j\omega_o t}$

where ω_o is a constant. Find the power spectral density of $Z(t)$. **[5 marks]**

[10] (CO4/K3)

05. A discrete time WSS random process $X[n]$ has zero mean and auto covariance function

$$C_{XX}[m] = \begin{cases} \sigma_x^2 & m = 0 \\ \alpha\sigma_x^2 & |m| = 1 \\ 0 & |m| \geq 2 \end{cases}$$

where $0 \leq \alpha \leq 1$

The process $X[n]$ is passed through a linear system characterized by the equation $Y[n] = X[n] - X[n-1]$

Find the auto covariance function $C_{YY}[m]$ and power spectral density $S_{XX}(e^{j\omega})$ of the output $Y(t)$.

[10] (CO4/K4)

06. Two identical networks are cascaded. Each has impulse response

$h_1(t) = u(t)3t\exp(-4t)$

$h_2(t) = u(t)4\exp(-5t)$

A WSS process X(t) is applied to the cascade's input

(i) Find the expression for the response $Y(t)$ of the cascade **[4 marks]**

(ii) If $E[X(t)] = \bar{X} = 6$, Find $\bar{Y}$ **[2 marks]**

(iii) Find $H(\omega)$ for the network **[4 marks]**

[10] (CO5/K3)

07. The auto correlation functions of a random signal X(t) and additive uncorrelated noise N(t) are

$R_{XX}(\tau) = \frac{5}{8}e^{-4|\tau|}$

$R_{NN}(\tau) = \frac{7}{6}e^{-3|\tau|}$

(i) Find the power spectrum of X(t) and N(t) **[5 marks]**

(ii) Find the transfer function of the Wiener filter for the given signal and noise **[5 marks]**

[10] (CO6/K3)

Section - II

Answer all Questions (2 × 15 Marks)

08. (a) A stationary random signal $X(t)$ has auto correlation function $R_{XX}(\tau) = 5\,e^{-2|\tau|}$. It is added to white noise [independent of $X(t)$] for $\frac{N_0}{2} = 10^{-3}$ and the sum is applied to filter having a transfer function

$H(\omega) = \frac{4\omega}{(2+j\omega)^3}$

Find the signal component of the output power spectrum and the average power in the output signal. **[8 marks]**

(b) An amplifier has a standard spot noise figure $F_0 = 6.31$. An engineer uses the amplifier to amplify the output of an antenna that is known to have an antenna temperature of $T_a = 180\,K$.

(i) What is the effective input noise temperature of the amplifier? **[4 marks]**

(ii) What is the operating spot noise figure? **[3 marks]**

[15] (CO6/K4)

09. (a) Two random variables X and Y are defined by $\overline{X} = 0, \overline{Y} = -2, \overline{X^2} = 2, \overline{Y^2} = 4$ and $R_{XY} = -2$. Two new random variables are defined as W and $U$are $W = 2X + Y$, $U = -X - 2Y$. Find $\overline{W}$, $\overline{U}$, $\overline{W^2}$, $\overline{U^2}$,σ_W^2 , σ_U^2, R_{UW}. **[10 Marks]**

(b) Two random variables X_1 and X_2 has zero means and variances $\sigma_{X_1}^2 = 9$ and $\sigma_{X_2}^2 = 16$. Their covariance $C_{X_1,X_2} = 5$. If X_1 and X_2 are linearly transformed to new variablesY_1 and Y_2 according to :

$Y_1 = 4X_1 - X_2$

$Y_2 = 2X_1 + 5X_2$

Find the mean, variance and covariance of $Y_1$and Y_2. **[5 Marks]**

[15] (CO2/K3)

BL-Bloom's Taxonomy Levels - (K1-Remembering,K2-Understanding,K3-Applying,K4-Analysing,K5-Evaluating,K6-Creating)