



VIT[®]

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

Reg No

Final Assessment Test(FAT) - Apr/May 2025

Programme	B.Tech.	Semester	Winter Semester 2024-25
Course Code	BMAT102L	Faculty Name	Prof. Mini Ghosh
Course Title	Differential Equations and Transforms	Slot	A1+TA1+TAA1
		Class Nbr	CH2024250500851
Time	3 hours	Max. Marks	100

Instructions To Candidates

- Write only your registration number in the designated box on the question paper. Writing anything elsewhere on the question paper will be considered a violation.

Course Outcomes

- CO1: Find solution for second and higher order differential equations, formation and solving partial differential equations
- CO2: Understand basic concepts of Laplace Transforms and solve problems with periodic functions, step functions, impulse functions and convolution
- CO3: Employ the tools of Fourier series and Fourier transforms
- CO4: Know the techniques of solving differential equations and partial differential equations
- CO5: Know the Z-transform and its application in population dynamics and digital signal processing

Answer any 10 Questions (10 × 10 Marks)

01. Solve the differential equation $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = \frac{e^t}{t^2 + 1}$ by using method of variation of parameters. [10] (CO1/K1)
02. (i) At $t = 0$, a current of 2 amperes flows in an RLC circuit with resistance $R = 40$ ohms, inductance $L = 2$ henrys and capacitance $C = 10^{-5}$ farads. Find the current flowing in the circuits at $t > 0$, if the initial charge on the capacitor is 1 coulomb. Assume that there is no e.m.f $E(t)$ for $t > 0$. (5 Marks)
- (ii) Form the partial differential equation by eliminating arbitrary function ϕ from $\phi(z^2 - xy, \frac{x}{z}) = 0$. (5 Marks) [10] (CO1/K2)
03. (i) Find the complete solution of a PDE: $9(p^2z + q^2) = 4$. (5 Marks)
- (ii) Solve $p^2 - 2pq + 3q = 5$. (5 Marks) [10] (CO1/K2)
04. (i) Evaluate the integral using Laplace transform $\int_0^\infty \frac{e^{-3t} - e^{-5t}}{t} dt$. (5 Marks)
- (ii) Find the inverse Laplace transform of $\frac{5s + 3}{(s - 1)(s^2 + 2s + 5)}$. (5 Marks) [10] (CO2/K3)
05. (i) Find the Laplace transform of the periodic function with period $2c$, given by
- $$f(x) = \begin{cases} t, & 0 < t < c \\ c - t, & c < t < 2c \end{cases} \quad (5 \text{ marks})$$
- (ii) Using the Fourier series of $f(x) = x$ in the interval $(0, 2\pi)$, show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$. (5 Marks) [10] (CO2,3/K1)
06. Solve the initial value problem $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} - 14y = 9 + u_3(t)$ given that $y(0) = 0$ and $y'(0) = 10$ using Laplace transform. Here ' $u_3(t)$ ' represents the unit step function.

[10] (CO4/K3)

07. Solve the first order partial differential equation $u_x(x, t) + u_t(x, t) + u(x, t) = 0$ with $u(0, t) = 0$ and $u(x, 0) = \sin x$ using Laplace transform. [10] (CO4/K3)
08. Find the half range Fourier cosine series of $f(x) = (\pi - x^2)$ in the interval $(0, \pi)$. Hence, find the sum of an infinite series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots$ [10] (CO3/K2)
09. Find the Fourier cosine transform of e^{-x^2} . Hence evaluate the Fourier sine transform of xe^{-x^2} . [10] (CO3/K2)
10. Find the Fourier transform of the function

$$f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}$$

Hence, show that $\int_0^\infty \frac{\cos \alpha x + \cos \alpha(x - \pi)}{1 - \alpha^2} d\alpha = \pi f(x)$.

[10] (CO3/K2)

11. (i) Find $Z[u_{n+2}]$ if $Z[u_n] = \frac{z}{z-1} + \frac{z}{z^2+1}$. (5 Marks)
 (ii) Find Z -transform of $(n+2)^2 + \sin \frac{n\pi}{4} + 10$. (5 Marks) [10] (CO5/K2)
12. Solve the following difference equation using Z -transform $u_{n+2} - 2u_{n+1} + u_n = 2^n$ with $u_0 = 2, u_1 = 1$. [10] (CO5/K3)

BL-Bloom's Taxonomy Levels - (K1-Remembering, K2-Understanding, K3-Applying, K4-Analysing, K5-Evaluating, K6-Creating)

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