VIT®

Vellore Institute of Technology

(Deemed to be University under section 3 of the UGC Act, 1956)

Reg. No.

Final Assessment Test(FAT) - Nov/Dec 2024

Programme	**B.Tech.**	Semester	**Fall Semester 2024-25**
Course Code	**BMAT101L**	Faculty Name	**Prof. Yuvarani C**
Course Title	**Calculus**	Slot	**B1+TB1**
		Class Nbr	**CH2024250103455**
Time	**3 hours**	Max. Marks	**100**

General Instructions

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

Course Outcomes

1. Apply single variable differentiation and integration to solve applied problems in engineering and find the maxima and minima of functions.
2. Evaluate partial derivatives, limits, total differentials, Jacobians, Taylor series and optimization problems involving several variables with or without constraints.
3. Evaluate multiple integrals in Cartesian, Polar, Cylindrical and Spherical coordinates.
4. Use special functions to evaluate various types of integrals.
5. Understand gradient, directional derivatives, divergence, curl, Green's, Stokes and Gauss Divergence theorems.

Section - I

Answer any 10 Questions (10 × 10 Marks)

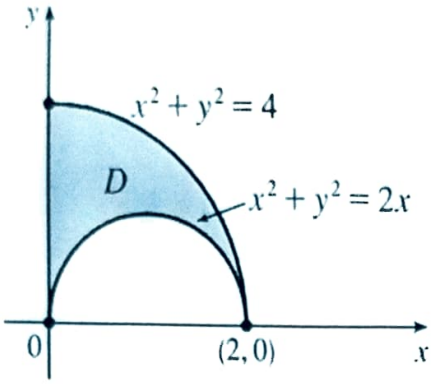
*M - Marks

Q.No	Question	*M	CO	BL
01.	Find the area of the region in the first quadrant bounded on the left by the y axis, below by the curve $y = \frac{x}{4}$, above left by the curve $y = 1 + \sqrt{x}$, and above right by the curve $y = \frac{2}{\sqrt{x}}$.	10	1	2
02.	a) Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{cosx}$, $y = 1$, $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$ about the x axis. (5 marks) b) Calculate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ for $u = \frac{2yz}{x}, v = \frac{3zx}{y}, w = \frac{4xy}{z}$. (5 marks)	10	1	1
03.	a) Given $w = ln(2x + 2y) + \tan(2x - 2y)$, find the value of $\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2}$. (5 marks) b) If $V = 2z^3 - 3z(x^2 + y^2)$, find $V_{xx} + V_{yy} + V_{zz}$. (5 marks)	10	2	2
04.	a) Find the Taylor series expansion of the function $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$ about the point $(1, 1)$ up to the second-degree terms. (5 marks) b) Express the following integral in terms of Gamma function: $\int_0^1 x^5(1 - x^3)^{10}\, dx$ (5 marks)	10	4	1
05.	Find the absolute maximum and minimum values of the function $f(x, y) = 4x^2 + 2y^2 + xy$ subject to the constraint $x + y \leq 4,\ x \geq 0,\ y \geq 0$.	10	2	3

06. 10 3 1

(a) Find the volume of the solid which is above the region in the XY-plane bounded by $x = 0; y = 0; x = \sqrt[3]{y}, x = 2$ and below the function $f(x, y) = e^{x^4}$. (5 marks)

(b) Evaluate $\int\int_D x\,dxdy$, where D is the region given as follows. (5 marks)



07. Evaluate the integral: 10 3 3

$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}}(x^2+y^2+z^2)^{3/2}\,dzdydx.$

08. a) Evaluate the integral $\iint dxdy$ over the first quadrant and below the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{1/3} = 1, a > 0, b > 0$. (7 marks) 10 4 2

b) Show that for smaller values of x

$erf(x) = \frac{2}{\pi}\left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \ldots\right)$. (3 marks)

09. a) Determine if the given vector field is conservative or not? 10 5 1

$\vec{F} = (18x^2 + 4z^3)\vec{i} - 12yz\vec{j} - (6y^2 - 12xz^2)\vec{k}$. (5 marks)

b) Find the directional derivative of the scalar function $\phi = xy + yz + zx$ at $(1, 2, 0)$ in the direction of $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$. Also, find its maximum value. (5 marks)

10. Prove that $\vec{F} = (y^2\cos x + z^3)\hat{i} + (2y\sin x - 4)\hat{j} + (3xz^2)\hat{k}$ is irrotational and find its scalar potential. 10 5 2

11. Verify Green's theorem for $\int_C (6y - 3y^2 + x)dx + yx^3dy$, where C is the curve formed by the upper half of the unit circle and the x axis. 10 5 1

12. If $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x + z)\hat{k}$, then evaluate $\oint_C \vec{F}.\vec{dr}$ using Stoke's theorem, where C is the boundary of a triangle with vertices (0,0,0), (1,0,0), and (1,1,0). 10 5 3

BL-Bloom's Taxonomy Levels - (1.Remembering, 2.Understanding, 3.Applying, 4.Analysing, 5.Evaluating, 6.Creating)