



VIT®

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

CHENNAI

Final Assessment Test(FAT) - Apr/May 2025

Programme	B.Tech.	Semester	Winter Semester 2024-25
Course Code	BMAT102L	Faculty Name	Prof. Dhivya M
Course Title	Differential Equations and Transforms	Slot	D1+TD1+TDD1
		Class Nbr	CH2024250500869
Time	3 hours	Max. Marks	100

Instructions To Candidates

- Write only your registration number in the designated box on the question paper. Writing anything elsewhere on the question paper will be considered a violation.

Course Outcomes

- CO1: Find solution for second and higher order differential equations, formation and solving partial differential equations.
- CO2: Understand basic concepts of Laplace Transforms and solve problems with periodic functions, step functions, impulse functions and convolution.
- CO3: Employ the tools of Fourier series and Fourier transforms.
- CO4: Know the techniques of solving differential equations and partial differential equations.
- CO5: Know the Z-transform and its application in population dynamics and digital signal processing.

Answer any 10 Questions (10 × 10 Marks)

01. Solve $(x+2)^2 \frac{d^2y}{dx^2} - (x+2) \frac{dy}{dx} + y = 3x+4$. [10] (CO1/K3)
02. (a) Find the charge on the capacitor in an RLC series circuit where $L = 5/3$ H, $R = 10\Omega$, $C = 1/30$ F, assuming zero initial charge and an initial current is 9 A. [5 Marks]
(b) Obtain a partial differential equation from $z = f(\sin(x) + \cos(y))$ by eliminating the arbitrary function. [5 Marks] [10] (CO1/K3)
03. Solve $x(y^2 + z)p + y(x^2 + z)q = z(x^2 - y^2)$. [10] (CO1/K3)
04. Evaluate $L^{-1} \left\{ \frac{5p^2 - 15p - 11}{(p+1)(p-2)^3} \right\}$, where $L^{-1}(F(p))$ represents the inverse Laplace transform. [10] (CO2/K3)
05. (a) Show that $L \left\{ \frac{\cos \sqrt{t}}{\sqrt{t}} \right\} = \sqrt{\frac{\pi}{p}} e^{-\frac{1}{4p}}$. [5 Marks]
(b) Find the half-range sine series of $f(x) = \sin x$. [5 Marks] [10] (CO2,3/K2)
06. Solve the simultaneous differential equations $\frac{dy}{dt} + 2x = \sin(2t)$, and $\frac{dx}{dt} + 2y = \cos(2t)$, $x(0) = 1$, $y(0) = 0$. [10] (CO4/K1)
07. Using Laplace transform, solve the first order partial differential equation $y_t = -y_x$, $x > 0$, $t > 0$ with $y(0, t) = 10$, and $y(x, 0) = 0$. [10] (CO4/K2)
08. Expand in Fourier series of $f(x) = x \sin(x)$ for $0 < x < 2\pi$ and deduce the result $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots = \frac{\pi-2}{4}$. [10] (CO3/K3)
09. (a) Find the function if its Fourier sine transform is $\frac{e^{-t}}{t}$. [5 Marks]
(b) Show that $e^{-x^2/2}$ is self-reciprocal with respect to the Fourier cosine transform. [5 Marks] [10] (CO3/K1)
10. Using the Fourier transform, solve $\frac{\partial u}{\partial t} = 9 \frac{\partial^2 u}{\partial x^2}$, $-\infty < x < \infty$, $t \geq 0$ subject to the condition $u(x, 0) = f(x)$. [10] (CO3/K3)

11. Solve the difference equation $y_{n+2} - 2 \cos(a)y_{n+1} + y_n = 0$ with $y_0 = 1, y_1 = \cos(a)$ using Z-transform. [10] (CO5/K3)
12. (a) Find $Z[u(n)]$ where $u(n) = (n+1)^3$. [5 Marks]
(b) Find $Z^{-1} \left[\frac{z^2}{(z-2)^3} \right]$. [5 Marks]

[10] (CO5/K3)

BL-Bloom's Taxonomy Levels - (K1-Remembering,K2-Understanding,K3-Applying,K4-Analysing,K5-Evaluating,K6-Creating)

