



VIT

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

Continuous Assessment Test (CAT) - I, August 2024

Programme	: B.Tech.	Semester	: Fall 2024-2025
Course Title	: Calculus	Code	: BMAT101L
Faculty	: Dr. SAGITHYA T Dr. SANKARSAN TARAI Dr. SANDIP SAHA Dr. VIJAY KUMAR P Ms. KIRUTHIKA Dr. NATHIYA N Dr. SUDIP DEBNATH Dr. SAURABH CHANDRA MAURY Dr. SOUMENDU ROY Ms. SRILEKHA	Slot	: E1+TE1
Duration	: 1 ½ Hours	Class No	: CH2024250103894 CH2024250103896 CH2024250103901 CH2024250103905 CH2024250103919 CH2024250103920 CH2024250103922 CH2024250103924 CH2024250103926 CH2024250103996
		Max. Marks	: 50

General Instructions:

- Write only your registration number on the question paper in the box provided and do not write other information.
- Only non-programmable calculator without storage is permitted.

PART A

Answer all the Questions (5 x 10 = 50 marks)

Q. No.	Question Description	Marks
1.	(a) Find all possible 'c' value(s) of Mean Value Theorem applied to the function $f(x) = 2x + \sqrt{x-1}$ on $[1,5]$. (b) Find absolute maximum and minimum for the function $f(x) = \sin(x) + \cos^2(x)$ in $[0, \pi]$.	5+5
2.	For the function $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x + 1$, (i) find the interval where $f(x)$ is increasing and decreasing, (ii) find where the local extrema occur, and (ii) find the intervals where $f(x)$ is concave up and concave down.	10
3.	(a) Determine the area between the two curves $y = x^2 - 2x$ and $y = x$ in the first quadrant. (b) Determine the volume of the solid obtained by rotating the region bounded by $y = x$, $y = 2x - x^2$, about the line $y = 0$ in the first quadrant.	5+5
4.	(a) Show that the function $f(x,y) = \begin{cases} \frac{x^4-y^4}{x^4+y^4}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$, is not continuous at the origin $(0,0)$.	2.5+ 2.5+ 5

(b) Show that for the function $f(x, y) = \begin{cases} y + x \sin\left(\frac{1}{y}\right), & \text{if } y \neq 0 \\ 0, & \text{if } y = 0 \end{cases}$, the $\lim_{y \rightarrow 0} \lim_{x \rightarrow 0} f(x, y)$ exist but

$\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y)$ does not exist.

(c) If $v = (x^2 - y^2) * f(xy)$, then prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = (x^4 - y^4)f''(xy)$.

5. (a) If $z = 2xy - 3x^2y$ and x is increasing at rate of 2 cm/s, determine at what rate y must be changing so that z shall be neither increasing nor decreasing at the instant when $x = 3$ cm and $y = 1$ cm.

4 + 6

(b) Determine whether $u = x^2 e^{-y} \cosh(z)$, $v = x^2 e^{-y} \sinh(z)$, $w = 3x^4 e^{-2y}$ are functionally dependent. If so, find the relation between them.